



ACHIEVE ACADEMY

GCSE Maths — Higher Tier

Surds · Practice Booklet

NAME: _____

CLASS: _____

DATE: _____

☐ Developing

☒ Secure

☐ Greater Depth

Answer every question in the working space provided. Method marks are available for correct working even if the final answer is wrong. Check the time allowed with your tutor before you start.

Surds

Total marks available: 103

● Surds

$$\sqrt{127} + 32\sqrt{7}$$

Q1 · ● Developing · 4 marks

METHOD

$$\begin{aligned} &= 5 \times 3 + 5 \times 4\sqrt{7} + 4\sqrt{7} \times 3 + 4\sqrt{7} \times 4\sqrt{7} \\ &= 15 + 20\sqrt{7} + 12\sqrt{7} + 16 \times 7 \\ &= (15 + 112) + (20 + 12)\sqrt{7} \\ &= 127 + 32\sqrt{7} \end{aligned}$$

$$\sqrt{15} + 8\sqrt{5}$$

Q2 · ● Developing · 3 marks

METHOD

$$\sqrt{5} \times 8 + \sqrt{5} \times 3\sqrt{5} = 8\sqrt{5} + 3 \times 5 = 15 + 8\sqrt{5}$$

$$\sqrt{58} + 20\sqrt{7}$$

Q3 · ● Developing · 4 marks

METHOD

$$\begin{aligned} &= 4 \times 4 + 4 \times 2\sqrt{7} + 3\sqrt{7} \times 4 + 3\sqrt{7} \times 2\sqrt{7} \\ &= 16 + 8\sqrt{7} + 12\sqrt{7} + 6 \times 7 \\ &= (16 + 42) + (8 + 12)\sqrt{7} \\ &= 58 + 20\sqrt{7} \end{aligned}$$

$$\sqrt{223} + 17\sqrt{11}$$

Q4 · ● Developing · 4 marks

METHOD

$$\begin{aligned} &= 1 \times 3 + 1 \times 5\sqrt{11} + 4\sqrt{11} \times 3 + 4\sqrt{11} \times 5\sqrt{11} \\ &= 3 + 5\sqrt{11} + 12\sqrt{11} + 20 \times 11 \\ &= (3 + 220) + (5 + 12)\sqrt{11} \\ &= 223 + 17\sqrt{11} \end{aligned}$$

$$\checkmark 204 + 21\sqrt{10}$$

Q5 · ● Developing · 4 marks

METHOD

$$\begin{aligned} &= 1 \times 4 + 1 \times 5\sqrt{10} + 4\sqrt{10} \times 4 + 4\sqrt{10} \times 5\sqrt{10} \\ &= 4 + 5\sqrt{10} + 16\sqrt{10} + 20 \times 10 \\ &= (4 + 200) + (5 + 16)\sqrt{10} \\ &= 204 + 21\sqrt{10} \end{aligned}$$

$$\checkmark 60 + 2\sqrt{10}$$

Q6 · ● Developing · 3 marks

METHOD

$$\sqrt{10} \times 2 + \sqrt{10} \times 6\sqrt{10} = 2\sqrt{10} + 6 \times 10 = 60 + 2\sqrt{10}$$

$$\checkmark 154 + 16\sqrt{10}$$

Q7 · ● Developing · 4 marks

METHOD

$$\begin{aligned} &= 2 \times 2 + 2 \times 5\sqrt{10} + 3\sqrt{10} \times 2 + 3\sqrt{10} \times 5\sqrt{10} \\ &= 4 + 10\sqrt{10} + 6\sqrt{10} + 15 \times 10 \\ &= (4 + 150) + (10 + 6)\sqrt{10} \\ &= 154 + 16\sqrt{10} \end{aligned}$$

$$\checkmark 165 + 45\sqrt{7}$$

Q8 · ● Developing · 4 marks

METHOD

$$\begin{aligned} &= 5 \times 5 + 5 \times 4\sqrt{7} + 5\sqrt{7} \times 5 + 5\sqrt{7} \times 4\sqrt{7} \\ &= 25 + 20\sqrt{7} + 25\sqrt{7} + 20 \times 7 \\ &= (25 + 140) + (20 + 25)\sqrt{7} \\ &= 165 + 45\sqrt{7} \end{aligned}$$

$$\checkmark 119 + 26\sqrt{13}$$

Q9 · ● Developing · 4 marks

METHOD

$$\begin{aligned} &= 3 \times 5 + 3 \times 2\sqrt{13} + 4\sqrt{13} \times 5 + 4\sqrt{13} \times 2\sqrt{13} \\ &= 15 + 6\sqrt{13} + 20\sqrt{13} + 8 \times 13 \\ &= (15 + 104) + (6 + 20)\sqrt{13} \\ &= 119 + 26\sqrt{13} \end{aligned}$$

$$\checkmark 9 + 2\sqrt{3}$$

Q10 · ● Developing · 3 marks

METHOD

$$\sqrt{3} \times 2 + \sqrt{3} \times 3\sqrt{3} = 2\sqrt{3} + 3 \times 3 = 9 + 2\sqrt{3}$$

$$\checkmark 12\sqrt{11}/11$$

Q11 · ● Secure · 3 marks

METHOD

$$12/\sqrt{11} = (12\sqrt{11})/11$$

$$\checkmark 4\sqrt{2}$$

Q12 · ● Secure · 3 marks

METHOD

$$8/\sqrt{2} = (8\sqrt{2})/2 = 4\sqrt{2}$$

$$\checkmark 2\sqrt{14}/7$$

Q13 · ● Secure · 3 marks

METHOD

$$4/\sqrt{14} = (4\sqrt{14})/14, \text{ which simplifies to } (2\sqrt{14})/7$$

$$\checkmark 1 + 3\sqrt{11}$$

Q14 · ● Secure · 3 marks

METHOD

$$\begin{aligned} &\text{Divide each term by 5: } (5 + 15\sqrt{11}) / 5 = 5/5 + (15/5)\sqrt{11} \\ &= 1 + 3\sqrt{11} \end{aligned}$$

$$\sqrt{4 + 3\sqrt{3}}$$

Q15 · ● Secure · 3 marks

METHOD

Divide each term by 3: $(12 + 9\sqrt{3}) / 3 = 12/3 + (9/3)\sqrt{3}$
 $= 4 + 3\sqrt{3}$

$$\sqrt{3\sqrt{2}/2}$$

Q16 · ● Secure · 3 marks

METHOD

$3/\sqrt{2} = (3\sqrt{2})/2$

$$\sqrt{3 + 2\sqrt{3}}$$

Q17 · ● Secure · 3 marks

METHOD

Divide each term by 4: $(12 + 8\sqrt{3}) / 4 = 12/4 + (8/4)\sqrt{3}$
 $= 3 + 2\sqrt{3}$

$$\sqrt{3 + 4\sqrt{2}}$$

Q18 · ● Secure · 3 marks

METHOD

Divide each term by 2: $(6 + 8\sqrt{2}) / 2 = 6/2 + (8/2)\sqrt{2}$
 $= 3 + 4\sqrt{2}$

$$\sqrt{\sqrt{3}}$$

Q19 · ● Secure · 3 marks

METHOD

$3/\sqrt{3} = (3\sqrt{3})/3 = \sqrt{3}$

$$\checkmark 4 + 3\sqrt{11}$$

Q20 · ● Secure · 3 marks

METHOD

Divide each term by 2: $(8 + 6\sqrt{11}) / 2 = 8/2 + (6/2)\sqrt{11}$
 $= 4 + 3\sqrt{11}$

$$\checkmark (3\sqrt{3} - 2) / 23$$

Q21 · ● Greater Depth · 4 marks

METHOD

Multiply top and bottom by the conjugate $(2 - 3\sqrt{3})$:
 $1/(2 + 3\sqrt{3}) \times (2 - 3\sqrt{3})/(2 - 3\sqrt{3})$
 $= (2 - 3\sqrt{3}) / (2^2 - 3^2 \times 3)$
 $= (3\sqrt{3} - 2) / 23$

$$\checkmark (2\sqrt{5} - 1) / 19$$

Q22 · ● Greater Depth · 4 marks

METHOD

Multiply top and bottom by the conjugate $(1 - 2\sqrt{5})$:
 $1/(1 + 2\sqrt{5}) \times (1 - 2\sqrt{5})/(1 - 2\sqrt{5})$
 $= (1 - 2\sqrt{5}) / (1^2 - 2^2 \times 5)$
 $= (2\sqrt{5} - 1) / 19$

$$\checkmark (4\sqrt{3} - 3) / 39$$

Q23 · ● Greater Depth · 4 marks

METHOD

Multiply top and bottom by the conjugate $(3 - 4\sqrt{3})$:
 $1/(3 + 4\sqrt{3}) \times (3 - 4\sqrt{3})/(3 - 4\sqrt{3})$
 $= (3 - 4\sqrt{3}) / (3^2 - 4^2 \times 3)$
 $= (4\sqrt{3} - 3) / 39$

$$\checkmark 12\sqrt{6} \text{ cm}$$

Q24 · ● Greater Depth · 3 marks

METHOD

Perimeter = $3\sqrt{6} + 6\sqrt{6} + 3\sqrt{6} = 12\sqrt{6} \text{ cm}$

✓ $9\sqrt{7}$ cm

Q25 · ● Greater Depth · 3 marks

METHOD

$$\text{Perimeter} = 3\sqrt{7} + 3\sqrt{7} + 3\sqrt{7} = 9\sqrt{7} \text{ cm}$$

✓ $10\sqrt{7}$ cm

Q26 · ● Greater Depth · 3 marks

METHOD

$$\text{Perimeter} = 3\sqrt{7} + 5\sqrt{7} + 2\sqrt{7} = 10\sqrt{7} \text{ cm}$$

✓ $15\sqrt{10}$ cm

Q27 · ● Greater Depth · 3 marks

METHOD

$$\text{Perimeter} = 4\sqrt{10} + 6\sqrt{10} + 5\sqrt{10} = 15\sqrt{10} \text{ cm}$$

✓ $(3\sqrt{10} - 4) / 74$

Q28 · ● Greater Depth · 4 marks

METHOD

Multiply top and bottom by the conjugate $(4 - 3\sqrt{10})$:

$$1/(4 + 3\sqrt{10}) \times (4 - 3\sqrt{10})/(4 - 3\sqrt{10})$$

$$= (4 - 3\sqrt{10}) / (4^2 - 3^2 \times 10)$$

$$= (3\sqrt{10} - 4) / 74$$

✓ $(4\sqrt{2} - 1) / 31$

Q29 · ● Greater Depth · 4 marks

METHOD

Multiply top and bottom by the conjugate $(1 - 4\sqrt{2})$:

$$1/(1 + 4\sqrt{2}) \times (1 - 4\sqrt{2})/(1 - 4\sqrt{2})$$

$$= (1 - 4\sqrt{2}) / (1^2 - 4^2 \times 2)$$

$$= (4\sqrt{2} - 1) / 31$$

$$\sqrt{(3\sqrt{6} - 5) / 29}$$

Q30 · ● Greater Depth · 4 marks

METHOD

Multiply top and bottom by the conjugate $(5 - 3\sqrt{6})$:

$$1/(5 + 3\sqrt{6}) \times (5 - 3\sqrt{6})/(5 - 3\sqrt{6})$$

$$= (5 - 3\sqrt{6}) / (5^2 - 3^2 \times 6)$$

$$= (3\sqrt{6} - 5) / 29$$