



ACHIEVE ACADEMY

GCSE Maths — Higher Tier

Non-Linear Simultaneous Equations · Practice Booklet

NAME: _____

CLASS: _____

DATE: _____

☐ Developing

☒ Secure

☐ Greater Depth

Answer every question in the working space provided. Method marks are available for correct working even if the final answer is wrong. Check the time allowed with your tutor before you start.

Non-Linear Simultaneous Equations

Total marks available: 134

● Non-Linear Simultaneous Equations

✓ $3t^2 - 6t = 0$

Q1 · ● Developing · 3 marks

METHOD

$$t - 2 = 3t^2 - 5t - 2$$

$$0 = 3t^2 - 5t - 2 + 2 - t$$

$$3t^2 - 6t = 0$$

✓ $3t^2 + 3t - 6 = 0$

Q2 · ● Developing · 3 marks

METHOD

$$t + 2 = 3t^2 + 4t - 4$$

$$0 = 3t^2 + 4t - 4 - 2 - t$$

$$3t^2 + 3t - 6 = 0$$

✓ $4a^2 + 4a - 11 = 0$

Q3 · ● Developing · 3 marks

METHOD

$$a + 5 = 4a^2 + 5a - 6$$

$$0 = 4a^2 + 5a - 6 - 5 - a$$

$$4a^2 + 4a - 11 = 0$$

✓ $(-2, -13)$ and $(6, 3)$

Q4 · ● Developing · 5 marks

METHOD

$$2t - 9 = t^2 - 2t - 21$$

$$0 = t^2 - 4t - 12$$

$$(t + 2)(t - 6) = 0$$

$$t = -2 \text{ or } t = 6$$

When $t = -2$, $y = -13$. When $t = 6$, $y = 3$.

✓ $(0, -7)$ and $(7, -35)$

Q5 · ● Developing · 5 marks

METHOD

$$-4t - 7 = t^2 - 11t - 7$$

$$0 = t^2 - 7t$$

$$(t)(t - 7) = 0$$

$$t = 0 \text{ or } t = 7$$

When $t = 0$, $y = -7$. When $t = 7$, $y = -35$.

✓ $3t^2 + 5t + 11 = 0$

Q6 · ● Developing · 3 marks

METHOD

$$t - 4 = 3t^2 + 6t + 7$$

$$0 = 3t^2 + 6t + 7 + 4 - t$$

$$3t^2 + 5t + 11 = 0$$

✓ $2a^2 - 7a = 0$

Q7 · ● Developing · 3 marks

METHOD

$$a - 4 = 2a^2 - 6a - 4$$

$$0 = 2a^2 - 6a - 4 + 4 - a$$

$$2a^2 - 7a = 0$$

✓ $(-3, -12)$ and $(2, 13)$

Q8 · ● Developing · 5 marks

METHOD

$$5t + 3 = t^2 + 6t - 3$$

$$0 = t^2 + t - 6$$

$$(t + 3)(t - 2) = 0$$

$$t = -3 \text{ or } t = 2$$

When $t = -3$, $y = -12$. When $t = 2$, $y = 13$.

✓(-7, -41) and (3, 19)

Q9 · ● Developing · 5 marks

METHOD

$$6a + 1 = a^2 + 10a - 20$$

$$0 = a^2 + 4a - 21$$

$$(a + 7)(a - 3) = 0$$

$$a = -7 \text{ or } a = 3$$

When $a = -7$, $y = -41$. When $a = 3$, $y = 19$.

✓(-4, -5) and (5, -14)

Q10 · ● Developing · 5 marks

METHOD

$$-a - 9 = a^2 - 2a - 29$$

$$0 = a^2 - a - 20$$

$$(a + 4)(a - 5) = 0$$

$$a = -4 \text{ or } a = 5$$

When $a = -4$, $y = -5$. When $a = 5$, $y = -14$.

✓(-4.27, 24.10) and (3.27, -6.10)

Q11 · ● Secure · 5 marks

METHOD

$$-4t + 7 = t^2 - 3t - 7$$

$$0 = t^2 + t - 14$$

$$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times (-14)}}{2}$$

$$t = \frac{-1 \pm \sqrt{57}}{2}$$

$$t = 3.27 \text{ or } t = -4.27$$

When $t = 3.27$, $y = -6.10$. When $t = -4.27$, $y = 24.10$.

✓(-3, -8) and (-2, -5)

Q12 · ● Secure · 5 marks

METHOD

$$3t + 1 = 2t^2 + 13t + 13$$

$$0 = 2t^2 + 10t + 12$$

$$2(t + 3)(t + 2) = 0$$

$$t = -3 \text{ or } t = -2$$

When $t = -3$, $y = -8$. When $t = -2$, $y = -5$.

✓(-2, 5) and (2, -3)

Q13 · ● Secure · 5 marks

METHOD

$$-2x + 1 = 3x^2 - 2x - 11$$

$$0 = 3x^2 - 12$$

$$3(x + 2)(x - 2) = 0$$

$$x = -2 \text{ or } x = 2$$

When $x = -2$, $y = 5$. When $x = 2$, $y = -3$.

✓(-0.46, -2.61) and (6.46, -23.39)

Q14 · ● Secure · 5 marks

METHOD

$$-3a - 4 = a^2 - 9a - 7$$

$$0 = a^2 - 6a - 3$$

$$a = \frac{-(-6) \pm \sqrt{((-6)^2 - 4 \times 1 \times (-3))}}{2}$$

$$a = \frac{6 \pm \sqrt{48}}{2}$$

$$a = 6.46 \text{ or } a = -0.46$$

When $a = 6.46$, $y = -23.39$. When $a = -0.46$, $y = -2.61$.

✓(-7.58, 26.75) and (1.58, -0.75)

Q15 · ● Secure · 5 marks

METHOD

$$-3t + 4 = t^2 + 3t - 8$$

$$0 = t^2 + 6t - 12$$

$$t = \frac{-(-6) \pm \sqrt{((-6)^2 - 4 \times 1 \times (-12))}}{2}$$

$$t = \frac{-6 \pm \sqrt{84}}{2}$$

$$t = 1.58 \text{ or } t = -7.58$$

When $t = 1.58$, $y = -0.75$. When $t = -7.58$, $y = 26.75$.

✓(-4.19, 12.77) and (1.19, -8.77)

Q16 · ● Secure · 5 marks

METHOD

$$-4x - 4 = x^2 - x - 9$$

$$0 = x^2 + 3x - 5$$

$$x = \frac{-(-3) \pm \sqrt{((-3)^2 - 4 \times 1 \times (-5))}}{2}$$

$$x = \frac{-3 \pm \sqrt{29}}{2}$$

$$x = 1.19 \text{ or } x = -4.19$$

When $x = 1.19$, $y = -8.77$. When $x = -4.19$, $y = 12.77$.

✓ $(-6, 28)$ and $(6, -32)$

Q17 · ● Secure · 5 marks

METHOD

$$-5x - 2 = 3x^2 - 5x - 110$$

$$0 = 3x^2 - 108$$

$$3(x + 6)(x - 6) = 0$$

$$x = -6 \text{ or } x = 6$$

When $x = -6$, $y = 28$. When $x = 6$, $y = -32$.

✓ (a) line drawn (see below); (b) $x = -1, y = -2$
and $x = 1, y = 2$

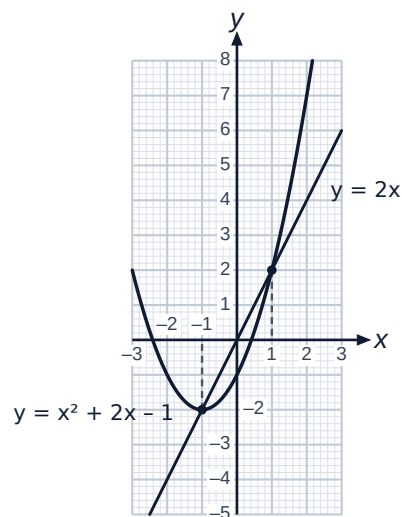
Q18 · ● Secure · 4 marks

METHOD

(a) The line $y = 2x$ passes through $(0, 0)$ with gradient 2. B1

(b) Read the points where the line crosses the curve: $(-1, -2)$; $(1, 2)$

M1 both points · A1 A1



✓ (a) line drawn (see below); (b) $x = -1, y = -2$
and $x = 2, y = 4$

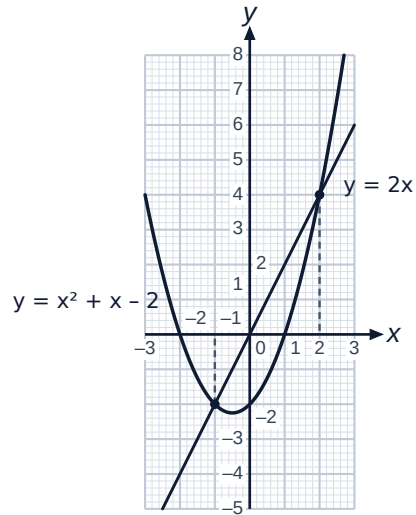
Q19 · ● Secure · 4 marks

METHOD

(a) The line $y = 2x$ passes through $(0, 0)$ with gradient 2 · B1

(b) Read the points where the line crosses the curve: $(-1, -2); (2, 4)$

M1 both points · A1 A1



✓ (a) line drawn (see below); (b) $x = -1.4, y = 1.4$
and $x = 1.4, y = -1.4$

Q20 · ● Secure · 4 marks

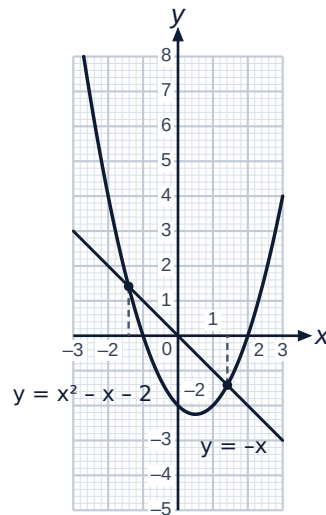
METHOD

(a) The line $y = -x$ passes through $(0, 0)$ with gradient -1 . B1

(b) Read the points where the line crosses the curve: $x = -1.4$ (accept -1.5 to -1.3), $y = 1.4$

(accept 1.3 to 1.5); $x = 1.4$ (accept 1.3 to 1.5), $y = -1.4$ (accept -1.5 to -1.3)

M1 both points · A1 A1



✓ Tangent at $(-1, -2)$

Q21 · ● Greater Depth · 5 marks

METHOD

$$3a + 1 = 4a^2 + 11a + 5$$

$$0 = 4a^2 + 8a + 4$$

$4(a + 1)^2 = 0$, a repeated root, so the line is a tangent

$$a = -1, y = -2$$

✓ Tangent at $(4, -22)$

Q22 · ● Greater Depth · 5 marks

METHOD

$$-5a - 2 = 3a^2 - 29a + 46$$

$$0 = 3a^2 - 24a + 48$$

$3(a - 4)^2 = 0$, a repeated root, so the line is a tangent

$$a = 4, y = -22$$

✓ Tangent at (5, 5)

Q23 · ● Greater Depth · 5 marks

METHOD

$$a = 3a^2 - 29a + 75$$

$$0 = 3a^2 - 30a + 75$$

$3(a - 5)^2 = 0$, a repeated root, so the line is a tangent

$$a = 5, y = 5$$

✓ (-0.14, -8.27) and (3.64, -0.73)

Q24 · ● Greater Depth · 5 marks

METHOD

$$2t - 8 = 2t^2 - 5t - 9$$

$$0 = 2t^2 - 7t - 1$$

$$t = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 2 \times (-1)}}{2 \times 2}$$

$$t = \frac{7 \pm \sqrt{57}}{4}$$

$$t = 3.64 \text{ or } t = -0.14$$

When $t = 3.64$, $y = -0.73$. When $t = -0.14$, $y = -8.27$.

✓ $x = -2.9, y = -2.8$ and $x = 0.5, y = 4.0$

Q25 · ● Greater Depth · 4 marks

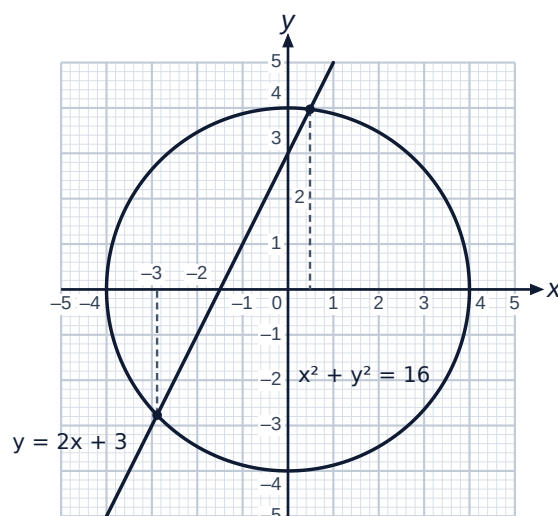
METHOD

The solutions are the two points where the line crosses the circle:

$x = -2.9$ (accept -3.0 to -2.8), $y = -2.8$ (accept -2.9 to -2.7)

$x = 0.5$ (accept 0.4 to 0.6), $y = 4.0$ (accept 3.9 to 4.1)

B1 B1 each point (both coordinates)



✓ $(-0.61, -2.04)$ and $(4.11, 21.54)$

Q26 · ● Greater Depth · 5 marks

METHOD

$$5a + 1 = 2a^2 - 2a - 4$$

$$0 = 2a^2 - 7a - 5$$

$$a = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 2 \times (-5)}}{2 \times 2}$$

$$a = \frac{7 \pm \sqrt{89}}{4}$$

$$a = 4.11 \text{ or } a = -0.61$$

When $a = 4.11$, $y = 21.54$. When $a = -0.61$, $y = -2.04$.

✓ $x = -0.9, y = -3.9$ and $x = 3.9, y = 0.9$

Q27 · ● Greater Depth · 4 marks

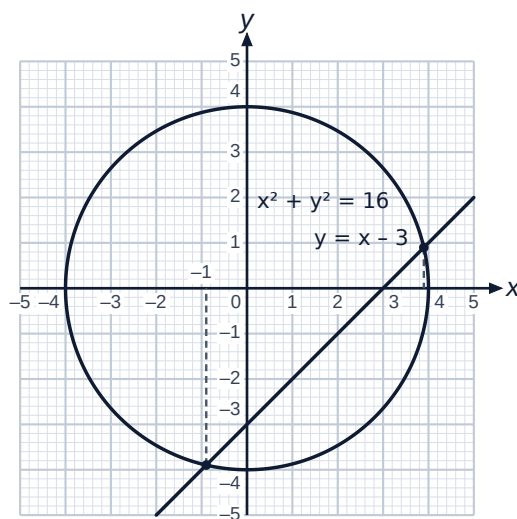
METHOD

The solutions are the two points where the line crosses the circle:

$x = -0.9$ (accept -1.0 to -0.8), $y = -3.9$ (accept -4.0 to -3.8)

$x = 3.9$ (accept 3.8 to 4.0), $y = 0.9$ (accept 0.8 to 1.0)

B1 B1 each point (both coordinates)



✓ Tangent at $(-2, 0)$

Q28 · ● Greater Depth · 5 marks

METHOD

$$-2a - 4 = 3a^2 + 10a + 8$$

$$0 = 3a^2 + 12a + 12$$

$3(a + 2)^2 = 0$, a repeated root, so the line is a tangent

$$a = -2, y = 0$$

✓ $(-1.48, 3.04)$ and $(3.15, 12.30)$

Q29 · ● Greater Depth · 5 marks

METHOD

$$2a + 6 = 3a^2 - 3a - 8$$

$$0 = 3a^2 - 5a - 14$$

$$a = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 3 \times (-14)}}{2 \times 3}$$

$$a = \frac{5 \pm \sqrt{193}}{6}$$

$$a = 3.15 \text{ or } a = -1.48$$

When $a = 3.15$, $y = 12.30$. When $a = -1.48$, $y = 3.04$.

✓ $x = -3.4, y = -3.7$ and $x = 5.0, y = 0.5$

Q30 · ● Greater Depth · 4 marks

METHOD

The solutions are the two points where the line crosses the circle:

$x = -3.4$ (accept -3.5 to -3.3), $y = -3.7$ (accept -3.8 to -3.6)

$x = 5.0$ (accept 4.9 to 5.1), $y = 0.5$ (accept 0.4 to 0.6)

B1 B1 each point (both coordinates)

