



# ACHIEVE ACADEMY

## GCSE Maths — Higher Tier

Iteration · Practice Booklet

NAME:

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CLASS:

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DATE:

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☐ Developing

☒ Secure

☐ Greater Depth

Answer every question in the working space provided. Method marks are available for correct working even if the final answer is wrong. Check the time allowed with your tutor before you start.

### Iteration

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Total marks available: 106

## ● Iteration

✓  $f(1) = -13, f(2) = 9$ , sign change  $\Rightarrow$  root in between

Q1 · ● Developing · 3 marks

### METHOD

When  $x = 1$ :  $4x^3 - 6x - 11 = -13$

When  $x = 2$ :  $4x^3 - 6x - 11 = 9$

The value changes from negative to positive, so there is a root between  $x = 1$  and  $x = 2$ .

✓  $x_2 = 3.168, x_3 = 3.037$

Q2 · ● Developing · 3 marks

### METHOD

$$x_2 = \sqrt[3]{(6 \times 3.8 + 9)} = 3.168$$

$$x_3 = \sqrt[3]{(6 \times 3.168 + 9)} = 3.037$$

✓  $x_2 = 2.884, x_3 = 2.688$

Q3 · ● Developing · 3 marks

### METHOD

$$x_2 = \sqrt[3]{(5 \times 3.8 + 5)} = 2.884$$

$$x_3 = \sqrt[3]{(5 \times 2.884 + 5)} = 2.688$$

✓  $x_2 = 2.819, x_3 = 2.702$

Q4 · ● Developing · 3 marks

### METHOD

$$x_2 = \sqrt[3]{(7 \times 3.2)} = 2.819$$

$$x_3 = \sqrt[3]{(7 \times 2.819)} = 2.702$$

✓  $f(1) = -7, f(2) = 2$ , sign change  $\Rightarrow$  root in between

Q5 · ● Developing · 3 marks

### METHOD

When  $x = 1$ :  $2x^3 - 5x - 4 = -7$

When  $x = 2$ :  $2x^3 - 5x - 4 = 2$

The value changes from negative to positive, so there is a root between  $x = 1$  and  $x = 2$ .

✓  $x_2 = 2.154, x_3 = 2.165$

Q6 · ● Developing · 3 marks

METHOD

$$x_2 = \sqrt[3]{(2.0 + 8)} = 2.154$$

$$x_3 = \sqrt[3]{(2.154 + 8)} = 2.165$$

✓  $f(1) = -13, f(2) = 13$ , sign change  $\Rightarrow$  root in between

Q7 · ● Developing · 3 marks

METHOD

When  $x = 1: 5x^3 - 9x - 9 = -13$

When  $x = 2: 5x^3 - 9x - 9 = 13$

The value changes from negative to positive, so there is a root between  $x = 1$  and  $x = 2$ .

✓  $x_2 = 1.866, x_3 = 1.630$

Q8 · ● Developing · 3 marks

METHOD

$$x_2 = \sqrt[3]{(5 \times 2.3 - 5)} = 1.866$$

$$x_3 = \sqrt[3]{(5 \times 1.866 - 5)} = 1.630$$

✓  $x_2 = 2.920, x_3 = 2.875$

Q9 · ● Developing · 3 marks

METHOD

$$x_2 = \sqrt[3]{(3 \times 3.3 + 15)} = 2.920$$

$$x_3 = \sqrt[3]{(3 \times 2.920 + 15)} = 2.875$$

✓  $x_2 = -1.560, x_3 = -2.232$

Q10 · ● Developing · 3 marks

METHOD

$$x_2 = \sqrt[3]{(2 \times 2.1 - 8)} = -1.560$$

$$x_3 = \sqrt[3]{(2 \times -1.560 - 8)} = -2.232$$

✓  $x_2 = 2.000, x_3 = 1.778, x_4 = 1.823$

Q11 · ● Secure · 4 marks

METHOD

$$x_2 = 16 \div (1 + 7) = 2.000$$

$$x_3 = 16 \div (2.000 + 7) = 1.778$$

$$x_4 = 16 \div (1.778 + 7) = 1.823$$

✓  $p = 1/2, q = 1/2$

Q12 · ● Secure · 3 marks

METHOD

$$4x^3 - 2x - 2 = 0$$

$$4x^3 = 2x + 2$$

$$x^3 = 1/2x + 1/2$$

$$x = \sqrt[3]{(1/2x + 1/2)}$$

✓  $x_2 = 4.500, x_3 = 1.636, x_4 = 3.414$

Q13 · ● Secure · 4 marks

METHOD

$$x_2 = 9 \div (1 + 1) = 4.500$$

$$x_3 = 9 \div (4.500 + 1) = 1.636$$

$$x_4 = 9 \div (1.636 + 1) = 3.414$$

✓  $x_2 = 1.000, x_3 = 1.500, x_4 = 1.333$

Q14 · ● Secure · 4 marks

METHOD

$$x_2 = 6 \div (3 + 3) = 1.000$$

$$x_3 = 6 \div (1.000 + 3) = 1.500$$

$$x_4 = 6 \div (1.500 + 3) = 1.333$$

✓  $p = 2/3, q = 14/3$

Q15 · ● Secure · 3 marks

METHOD

$$3x^3 - 2x - 14 = 0$$

$$3x^3 = 2x + 14$$

$$x^3 = 2/3x + 14/3$$

$$x = \sqrt[3]{(2/3x + 14/3)}$$

✓  $p = 3, q = 1$

Q16 · ● Secure · 3 marks

METHOD

$$2x^3 - 6x - 2 = 0$$

$$2x^3 = 6x + 2$$

$$x^3 = 3x + 1$$

$$x = \sqrt[3]{(3x + 1)}$$

✓  $x_2 = 1.667, x_3 = 1.875, x_4 = 1.739$

Q17 · ● Secure · 4 marks

METHOD

$$x_2 = 5 \div (2 + 1) = 1.667$$

$$x_3 = 5 \div (1.667 + 1) = 1.875$$

$$x_4 = 5 \div (1.875 + 1) = 1.739$$

✓  $x_2 = 1.875, x_3 = 2.182, x_4 = 2.089$

Q18 · ● Secure · 4 marks

METHOD

$$x_2 = 15 \div (3 + 5) = 1.875$$

$$x_3 = 15 \div (1.875 + 5) = 2.182$$

$$x_4 = 15 \div (2.182 + 5) = 2.089$$

✓  $x_2 = 0.875, x_3 = 1.806, x_4 = 1.456$

Q19 · ● Secure · 4 marks

METHOD

$$x_2 = 7 \div (5 + 3) = 0.875$$

$$x_3 = 7 \div (0.875 + 3) = 1.806$$

$$x_4 = 7 \div (1.806 + 3) = 1.456$$

✓  $p = 11/3, q = 8/3$

Q20 · ● Secure · 3 marks

METHOD

$$3x^3 - 11x - 8 = 0$$

$$3x^3 = 11x + 8$$

$$x^3 = 11/3x + 8/3$$

$$x = \sqrt[3]{(11/3x + 8/3)}$$

$$\checkmark x_2 = 2.321, x_3 = 2.327, x_4 = 2.329$$

Q21 · ● Greater Depth · 4 marks

METHOD

$$x_2 = \sqrt[3]{(5 \times 2.3 + 1)} = 2.321$$

$$x_3 = \sqrt[3]{(5 \times 2.321 + 1)} = 2.327$$

$$x_4 = \sqrt[3]{(5 \times 2.327 + 1)} = 2.329$$

$$\checkmark x = 1.41$$

Q22 · ● Greater Depth · 4 marks

METHOD

$$x_2 = \sqrt[3]{(2 \times 2.000)} = 1.587$$

$$x_3 = \sqrt[3]{(2 \times 1.587)} = 1.470$$

$$x_4 = \sqrt[3]{(2 \times 1.470)} = 1.432$$

$$x_5 = \sqrt[3]{(2 \times 1.432)} = 1.420$$

$$x_6 = \sqrt[3]{(2 \times 1.420)} = 1.416$$

$$x_7 = \sqrt[3]{(2 \times 1.416)} = 1.415$$

The value settles at 1.41 to 2 decimal places.

$$\checkmark x = -1.32$$

Q23 · ● Greater Depth · 4 marks

METHOD

$$x_2 = \sqrt[3]{(1.000 - 1)} = 0.000$$

$$x_3 = \sqrt[3]{(0.000 - 1)} = -1.000$$

$$x_4 = \sqrt[3]{(-1.000 - 1)} = -1.260$$

$$x_5 = \sqrt[3]{(-1.260 - 1)} = -1.312$$

$$x_6 = \sqrt[3]{(-1.312 - 1)} = -1.322$$

$$x_7 = \sqrt[3]{(-1.322 - 1)} = -1.324$$

The value settles at -1.32 to 2 decimal places.

$$\sqrt{x} = 2.17$$

Q24 · ● Greater Depth · 4 marks

METHOD

$$x_2 = \sqrt[3]{(1,000 + 8)} = 2.080$$

$$x_3 = \sqrt[3]{(2,080 + 8)} = 2.160$$

$$x_4 = \sqrt[3]{(2,160 + 8)} = 2.166$$

$$x_5 = \sqrt[3]{(2,166 + 8)} = 2.166$$

$$x_6 = \sqrt[3]{(2,166 + 8)} = 2.166$$

The value settles at 2.17 to 2 decimal places.

$$\sqrt{x_2} = 1.270, x_3 = 1.314, x_4 = 1.323$$

Q25 · ● Greater Depth · 4 marks

METHOD

$$x_2 = \sqrt[3]{(1.05 + 1)} = 1.270$$

$$x_3 = \sqrt[3]{(1.270 + 1)} = 1.314$$

$$x_4 = \sqrt[3]{(1.314 + 1)} = 1.323$$

$$\sqrt{x} = 1.90$$

Q26 · ● Greater Depth · 4 marks

METHOD

$$x_2 = \sqrt[3]{(1,000 + 5)} = 1.817$$

$$x_3 = \sqrt[3]{(1,817 + 5)} = 1.896$$

$$x_4 = \sqrt[3]{(1,896 + 5)} = 1.903$$

$$x_5 = \sqrt[3]{(1,903 + 5)} = 1.904$$

The value settles at 1.90 to 2 decimal places.

$$\sqrt{x} = 2.86$$

Q27 · ● Greater Depth · 4 marks

METHOD

$$x_2 = \sqrt[3]{(5 \times 1,000 + 9)} = 2.410$$

$$x_3 = \sqrt[3]{(5 \times 2,410 + 9)} = 2.761$$

$$x_4 = \sqrt[3]{(5 \times 2,761 + 9)} = 2.836$$

$$x_5 = \sqrt[3]{(5 \times 2,836 + 9)} = 2.851$$

$$x_6 = \sqrt[3]{(5 \times 2,851 + 9)} = 2.854$$

$$x_7 = \sqrt[3]{(5 \times 2,854 + 9)} = 2.855$$

The value settles at 2.86 to 2 decimal places.

$$\checkmark x = 2.75$$

Q28 · ● Greater Depth · 4 marks

METHOD

$$x_2 = \sqrt[3]{(5 \times 3.000 + 7)} = 2.802$$

$$x_3 = \sqrt[3]{(5 \times 2.802 + 7)} = 2.759$$

$$x_4 = \sqrt[3]{(5 \times 2.759 + 7)} = 2.750$$

$$x_5 = \sqrt[3]{(5 \times 2.750 + 7)} = 2.748$$

$$x_6 = \sqrt[3]{(5 \times 2.748 + 7)} = 2.747$$

The value settles at 2.75 to 2 decimal places.

$$\checkmark x_2 = -1.174, x_3 = -2.522, x_4 = -2.890$$

Q29 · ● Greater Depth · 4 marks

METHOD

$$x_2 = \sqrt[3]{(6 \times 1.23 - 9)} = -1.174$$

$$x_3 = \sqrt[3]{(6 \times -1.174 - 9)} = -2.522$$

$$x_4 = \sqrt[3]{(6 \times -2.522 - 9)} = -2.890$$

$$\checkmark x_2 = -1.917, x_3 = -2.218, x_4 = -2.239$$

Q30 · ● Greater Depth · 4 marks

METHOD

$$x_2 = \sqrt[3]{(1.95 - 9)} = -1.917$$

$$x_3 = \sqrt[3]{(-1.917 - 9)} = -2.218$$

$$x_4 = \sqrt[3]{(-2.218 - 9)} = -2.239$$