



ACHIEVE ACADEMY

GCSE Maths — Higher Tier

Algebraic Proof · Practice Booklet

NAME: _____

CLASS: _____

DATE: _____

☐ Developing

☒ Secure

☐ Greater Depth

Answer every question in the working space provided. Method marks are available for correct working even if the final answer is wrong. Check the time allowed with your tutor before you start.

Algebraic Proof

Total marks available: 130

● Algebraic Proof

✓ True (proven algebraically).

Q1 · ● Developing · 4 marks

METHOD

If m is even, m is a factor of 2, so $m(m + 1)$ is even.

If m is odd, then since 1 is odd, $m + 1$ is even, so $m(m + 1)$ is even.

Either way, $m(m + 1)$ is always even.

✓ True (proven algebraically).

Q2 · ● Developing · 4 marks

METHOD

$$(2t + 4)^2 - (2t)^2$$

$$= 4t^2 + 16t + 16 - 4t^2$$

$$= 16t + 16$$

$$= 4(4t + 4)$$

This is $4 \times$ an integer, so the difference is always a multiple of 4.

✓ True (proven algebraically).

Q3 · ● Developing · 4 marks

METHOD

If n is even, n is a factor of 2, so $n(n + 1)$ is even.

If n is odd, then since 1 is odd, $n + 1$ is even, so $n(n + 1)$ is even.

Either way, $n(n + 1)$ is always even.

✓ True (proven algebraically).

Q4 · ● Developing · 4 marks

METHOD

$$(2p + 8)^2 - (2p)^2$$

$$= 4p^2 + 32p + 64 - 4p^2$$

$$= 32p + 64$$

$$= 4(8p + 16)$$

This is $4 \times$ an integer, so the difference is always a multiple of 4.

✓ True (proven algebraically).

Q5 · ● Developing · 4 marks

METHOD

If m is even, m is a factor of 2, so $m(m + 3)$ is even.

If m is odd, then since 3 is odd, $m + 3$ is even, so $m(m + 3)$ is even.

Either way, $m(m + 3)$ is always even.

✓ True (proven algebraically).

Q6 · ● Developing · 4 marks

METHOD

$$\begin{aligned}(2p + 10)^2 - (2p)^2 \\&= 4p^2 + 40p + 100 - 4p^2 \\&= 40p + 100 \\&= 4(10p + 25)\end{aligned}$$

This is $4 \times$ an integer, so the difference is always a multiple of 4.

✓ True (proven algebraically).

Q7 · ● Developing · 4 marks

METHOD

If t is even, t is a factor of 2, so $t(t + 3)$ is even.

If t is odd, then since 3 is odd, $t + 3$ is even, so $t(t + 3)$ is even.

Either way, $t(t + 3)$ is always even.

✓ True (proven algebraically).

Q8 · ● Developing · 4 marks

METHOD

$$\begin{aligned}(2t + 10)^2 - (2t)^2 \\&= 4t^2 + 40t + 100 - 4t^2 \\&= 40t + 100 \\&= 4(10t + 25)\end{aligned}$$

This is $4 \times$ an integer, so the difference is always a multiple of 4.

✓ True (proven algebraically).

Q9 · ● Developing · 4 marks

METHOD

If p is even, p is a factor of 2, so $p(p + 3)$ is even.

If p is odd, then since 3 is odd, $p + 3$ is even, so $p(p + 3)$ is even.

Either way, $p(p + 3)$ is always even.

✓ True (proven algebraically).

Q10 · ● Developing · 4 marks

METHOD

If p is even, p is a factor of 2, so $p(p + 5)$ is even.

If p is odd, then since 5 is odd, $p + 5$ is even, so $p(p + 5)$ is even.

Either way, $p(p + 5)$ is always even.

✓ True (proven algebraically).

Q11 · ● Secure · 4 marks

METHOD

Let the two consecutive odd numbers be $2n + 1$ and $2n + 3$.

$$(2n + 1)(2n + 3) + 1$$

$$= 4n^2 + 8n + 3 + 1$$

$$= 4n^2 + 8n + 4$$

$$= 4(n^2 + 2n + 1)$$

This is $4 \times$ an integer, so it is always a multiple of 4.

✓ True (proven algebraically).

Q12 · ● Secure · 4 marks

METHOD

$$(t + 4)^2 - (t - 4)^2$$

$$= (t^2 + 8t + 16) - (t^2 - 8t + 16)$$

$$= 16t$$

This is $16 \times$ an integer, so it is always a multiple of 16.

✓ True (proven algebraically).

Q13 · ● Secure · 4 marks

METHOD

$$\begin{aligned}(t + 6)^2 - (t - 6)^2 \\&= (t^2 + 12t + 36) - (t^2 - 12t + 36) \\&= 24t\end{aligned}$$

This is $24 \times$ an integer, so it is always a multiple of 24.

✓ True (proven algebraically).

Q14 · ● Secure · 4 marks

METHOD

$$\begin{aligned}(m + 3)^2 - (m - 3)^2 \\&= (m^2 + 6m + 9) - (m^2 - 6m + 9) \\&= 12m\end{aligned}$$

This is $12 \times$ an integer, so it is always a multiple of 12.

✓ True (proven algebraically).

Q15 · ● Secure · 4 marks

METHOD

$$\begin{aligned}(p + 6)^2 - (p - 6)^2 \\&= (p^2 + 12p + 36) - (p^2 - 12p + 36) \\&= 24p\end{aligned}$$

This is $24 \times$ an integer, so it is always a multiple of 24.

✓ True (proven algebraically).

Q16 · ● Secure · 4 marks

METHOD

$$\begin{aligned}(m + 2)^2 - (m - 2)^2 \\&= (m^2 + 4m + 4) - (m^2 - 4m + 4) \\&= 8m\end{aligned}$$

This is $8 \times$ an integer, so it is always a multiple of 8.

✓ True (proven algebraically).

Q17 · ● Secure · 4 marks

METHOD

$$\begin{aligned}(n+6)^2 - (n-6)^2 \\&= (n^2 + 12n + 36) - (n^2 - 12n + 36) \\&= 24n\end{aligned}$$

This is $24 \times$ an integer, so it is always a multiple of 24.

✓ True (proven algebraically).

Q18 · ● Secure · 4 marks

METHOD

$$\begin{aligned}(n+3)^2 - (n-3)^2 \\&= (n^2 + 6n + 9) - (n^2 - 6n + 9) \\&= 12n\end{aligned}$$

This is $12 \times$ an integer, so it is always a multiple of 12.

✓ True (proven algebraically).

Q19 · ● Secure · 4 marks

METHOD

$$\begin{aligned}(p+4)^2 - (p-4)^2 \\&= (p^2 + 8p + 16) - (p^2 - 8p + 16) \\&= 16p\end{aligned}$$

This is $16 \times$ an integer, so it is always a multiple of 16.

✓ True (proven algebraically).

Q20 · ● Secure · 4 marks

METHOD

$$\begin{aligned}(t+7)^2 - (t-7)^2 \\&= (t^2 + 14t + 49) - (t^2 - 14t + 49) \\&= 28t\end{aligned}$$

This is $28 \times$ an integer, so it is always a multiple of 28.

✓ $2/(p(p+2))$

Q21 · ● Greater Depth · 5 marks

METHOD

$$\begin{aligned} & 1/p - 1/(p+2) \\ &= (p+2)/(p(p+2)) - p/(p(p+2)) \\ &= ((p+2) - p) / (p(p+2)) \\ &= 2 / (p(p+2)) \end{aligned}$$

✓ True (proven algebraically).

Q22 · ● Greater Depth · 5 marks

METHOD

The 3 consecutive even numbers are $2t$, $2t+2$, ..., $2t+4$.

$$\text{Sum} = 2t + (2t+2) + (2t+4)$$

$$= 6t + 6$$

$$= 6(t+1)$$

This is $6 \times$ an integer, so the sum is always a multiple of 6.

✓ $7/(t(t+7))$

Q23 · ● Greater Depth · 5 marks

METHOD

$$\begin{aligned} & 1/t - 1/(t+7) \\ &= (t+7)/(t(t+7)) - t/(t(t+7)) \\ &= ((t+7) - t) / (t(t+7)) \\ &= 7 / (t(t+7)) \end{aligned}$$

✓ True (proven algebraically).

Q24 · ● Greater Depth · 5 marks

METHOD

The 3 consecutive even numbers are $2n$, $2n+2$, ..., $2n+4$.

$$\text{Sum} = 2n + (2n+2) + (2n+4)$$

$$= 6n + 6$$

$$= 6(n+1)$$

This is $6 \times$ an integer, so the sum is always a multiple of 6.

✓ $6/(m(m+6))$

Q25 · ● Greater Depth · 5 marks

METHOD

$$\begin{aligned} & 1/m - 1/(m+6) \\ &= (m+6)/(m(m+6)) - m/(m(m+6)) \\ &= ((m+6) - m) / (m(m+6)) \\ &= 6 / (m(m+6)) \end{aligned}$$

✓ $6/(p(p+6))$

Q26 · ● Greater Depth · 5 marks

METHOD

$$\begin{aligned} & 1/p - 1/(p+6) \\ &= (p+6)/(p(p+6)) - p/(p(p+6)) \\ &= ((p+6) - p) / (p(p+6)) \\ &= 6 / (p(p+6)) \end{aligned}$$

✓ $8/(p(p+8))$

Q27 · ● Greater Depth · 5 marks

METHOD

$$\begin{aligned} & 1/p - 1/(p+8) \\ &= (p+8)/(p(p+8)) - p/(p(p+8)) \\ &= ((p+8) - p) / (p(p+8)) \\ &= 8 / (p(p+8)) \end{aligned}$$

✓ True (proven algebraically).

Q28 · ● Greater Depth · 5 marks

METHOD

The 7 consecutive even numbers are $2n, 2n+2, \dots, 2n+12$.

$$\begin{aligned} \text{Sum} &= 2n + (2n+2) + (2n+4) + (2n+6) + (2n+8) + (2n+10) + (2n+12) \\ &= 14n + 42 \\ &= 14(n+3) \end{aligned}$$

This is $14 \times$ an integer, so the sum is always a multiple of 14.

✓ $5/(t(t+5))$

Q29 · ● Greater Depth · 5 marks

METHOD

$$\begin{aligned} & 1/t - 1/(t+5) \\ &= (t+5)/(t(t+5)) - t/(t(t+5)) \\ &= ((t+5) - t) / (t(t+5)) \\ &= 5 / (t(t+5)) \end{aligned}$$

✓ True (proven algebraically).

Q30 · ● Greater Depth · 5 marks

METHOD

The 5 consecutive even numbers are $2p, 2p+2, \dots, 2p+8$.

$$\text{Sum} = 2p + (2p+2) + (2p+4) + (2p+6) + (2p+8)$$

$$= 10p + 20$$

$$= 10(p+2)$$

This is $10 \times$ an integer, so the sum is always a multiple of 10.